Hybrid effects on vehicle stability

Ammendment
Earlier editions of this paper contained a formula from reference 1 for calculating the moment of inertia of a liquid propellant in a cylindrical tank. This formula was in error, and has been removed from this edition. A correct formula is now in our paper 'Mass properties of a hybrid's liquid propellant'.

Introduction
It's now been several years since the rocketry world started launching with impunity kits designed for solid propulsion, but with a hybrid motor inserted up them instead.

So far, there have been surprisingly few reported problems.
Or rather, it hasn't been realised that the new crop of weird things that occasionally happen may have been caused by the hybrid.

So what are the extra effects to take note of?

Words in **bold** appear in the Glossary at the end of the document.

Centre of Gravity shift
First off, there's the fairly obvious point that the Centre of Gravity (CG) of a rocket vehicle powered by a solid motor moves nosewards during the burn as it ejects propellant mass that was in a motor that typically was aft of the CG.
The vehicle ends up a lot more stable at burnout than it was at launch.

The vehicle's **Barrowman** static stability (in **Calibers**) has the most noticable effect on the vehicle's trajectory where the vehicle is travelling at low speed: launch and apogee.
Stability at launch is the one we all know about, and we tune the **Calibers** accordingly.
At apogee, the vehicle is much more stable, so there's no need to bother worrying about stability there.

Depending on the particular hybrid system, the vehicle CG can actually track tailwards during the burn, because there's all that oxidiser mass pouring out of a tank that is ahead of the CG.
In moderation (i.e provided it doesn't go unstable), this is a useful effect, because the ensuing trajectory remains more vertical: the length of the moment arm between CG and Centre of Pressure (CP) is smaller, providing gravity with less torque to turn the trajectory over.

But the stability at apogee is a lot less than you'd get with a solid motor.
On the first flight of Aspire's FLARE hybrid vehicle, it was hit by a windshear just before apogee as it cleared the wind-shadow of a nearby ridge.
It tumbled, reducing apogee height, and this could have affected drogue deployment if there had been a drogue deployed at apogee.

Aspire now sim the CG over the whole burn to check that it doesn't dip too low at any point, and increase the launch static margin if we have to. This does require a long launchrail to keep the trajectory straight just after launch, but this is a small price to pay for a clean flight.

To estimate the CG shift during the burn, assume that the flowrate of nitrous out of the tank (and therefore the speed at which the surface of the liquid is falling in the tank) is directly proportional to the instantaneous thrust.
This reflects the fact that both the thrust and oxidiser flowrate fall as the tank pressure falls during the burn, and for nitrous at least, reflects that fact that the performance is fairly insensitve to the changing mixture ratio of fuel to oxidiser during the burn.

So a graph of the speed that the level in the tank is dropping versus time is pretty much the same shape as the thrustcurve.

Scale this level-dropping speed so that the tank just empties at burnout.

If you have accurate mass flow rate data for the nitrous, then do use that in preference, but bear in mind that some of the nitrous at the surface vaporises as the tank empties so the level falls faster than the mass continuity equation would predict.

(See our paper on nitrous oxide for details.)

Integrate this level-dropping speed with time to get the level of the fluid with time during the burn. (e.g. new level = previous level - speed times dt, where dt is, say, a quarter-second time interval.)

The mass of the fluid is then its density times its volume (= tank cross-sectional area times fluid height), and the fluid's own CG is then in the middle of the column of fluid, (i.e. at half the height of the fluid.)

Remember that the nitrous vapour in the tank is almost as dense as the liquid nitrous below it on a hot day; you can't ignore the mass of the vapour.

It occupies the entire space above the level of the liquid, and again, its own CG is pretty much in the middle. (i.e. halfway down from the top of the tank to the height of the fluid surface.)

Low thrust

Hybrid engines, especially home-made ones, tend to have low thrust at takeoff, and of course don't have a boost thrust at takeoff as solid motors do.

This can cause a trajectory that quickly goes horizontal, which tends to get a low apogee, and rips the 'chute off due to a high apogee airspeed.

Use a tall launch tower/rail for hybrids, and add solid booster motors if required to get the thrust-to-weight ratio higher at launch.

Large Polar Moment of Inertia

The polar axes are the pitch and yaw axes, not the longitudinal (roll) axis.

The long, heavy tanks supplied with commercial hybrid systems give the vehicle a high Moment of inertia about its pitch/roll axes.

This can make hybrid vehicles prone to **Roll resonance**, because the weathercock frequency reduces as the polar Moment of Inertia increases, low enough to hit the roll rate. (see glossary.)

Aspire's early FLARE rocket utilised a particularly heavy paintball tank.

The 2nd FLARE flight spun-up in roll due to an aerodynamic fairing that wasn't alligned properly, and hit roll resonance just after launch.

FLARE eventually gained enough airspeed to leave resonance: it stopped corkscrewing, but the trajectory had been skewed from the near-vertical launch into a long, low path with the usual excessive apogee airspeed that broke the recovery system.

FLARE was ignominiously dug-out with a spade.

…of the liquid

Here's food for thought: what's the polar moment of inertia of a liquid? (the moment of inertia of the nitrous about the *roll* axis is effectively zero.)

This is a really thorny little problem that has occupied some of science's best minds, and as the mass of oxidiser in a good hybrid system should be a large part of the launch mass, it strongly affects the dynamic stability, i.e. the response of the vehicle to gusts.

Some trajectory sims that model these dynamic effects require you to input the moment of inertias of the vehicle's components; what's the correct value for the liquid within the hybrid's oxidiser tank?

The moment of inertias of the parts of the vehicle that are solid are easily calculated using standard equations, but the nitrous clearly isn't solid.

Observation of different shapes of clear-plastic bottles of water being rotated by hand shows the main effects.

Far from the axis of rotation, the fluid is carried round by the walls of the bottle as the bottle rotates, so the moment of inertia here is almost as much as if the fluid were frozen in the bottle, i.e. as if it were solid.

The fluid near the axis of rotation of the bottle however, a roughly spherical region of the same diameter as the bottle, isn't affected by the bottle's rotation.

As the bottle rotates, this roughly spherical shape doesn't rotate, as if it were a free-to-rotate solid sphere on a low-friction pivot.

So this central region effectively has zero moment of inertia.

Between these two regions, the effects merge from one to the other: partial rotation.

If the tank is long and thin then the tank diameter is small in comparison to the scale of the tank, so the central non-rotating region of fluid is small; the tank's moment of inertia is almost as large as if the fluid was completely frozen solid.

But if the tank is squat, or the axis of rotation of the tank is near the base of the tank and the tank is almost empty, then most of the fluid is not rotating, so the moment of inertia is very much lower than the value it would be if the fluid were frozen solid.

See our paper 'Mass properties of a hybrid's liquid propellant' for a mathematical model to calculate the moment of inertia of the liquid in a tank.

Note that though the nitrous vapour in the tank is almost as dense as the liquid nitrous below it on a hot day, it is a vapour and not a liquid, so I've assumed that it has zero moment of inertia.

This may well be too much of an assumption, comments please.

As for the moment of inertia of the rest of the hybrid system including the empty tank, contact the manufacturer, or measure it.

Slosh

The analysis of the moment of inertia of the liquid in the tank doesn't care whether the tank is completely full or not: whether the tank has a lid on it doesn't affect the result.

But if the tank has no lid, or if it isn't completely full, then waves will slosh around on the surface of the liquid as the vehicle moves about.

The weight (mass) of these waves hitting the sides of the tank causes lateral forces on the vehicle that upset the trajectory.

Fortunately, the mass of the waves compared to the mass of the whole vehicle depends on the tank internal diameter; for vehicles a lot smaller than those that could launch a satellite, the effects of slosh can be ignored.

For example, Aspire's ADV2b 4-inch diameter vehicle uses a tank of similar diameter to the larger Hypertech hybrid tanks. From reference 2 I've calculated that the masses of the sloshing waves are just over one percent of the vehicle's lift-off mass.

Reference 4 gives a guide to calculating the moment of inertia of a liquid in a large tank that requires internal tank baffles to reduce slosh.

Glossary:

Barrowman:
The basic stability of fixed-fin rocket vehicles has been covered by James Barrowman and others, and won't be repeated here.
(Note that several books and web-pages contains errors in their reprinting of Barrowman's stability equations; it's better to get a copy of the original paper.)

Barrowman's method is a classic static-stability analysis: it simply tells you whether your fins are large enough so that your vehicle has a tendency to keep the nose pointing in the direction of flight as required, and it asumes that the ensuing rotation of the vehicle about it's CG is slow enough not to affect the analysis.

Calibers, Calibres:
In rocketry, vehicle dimensions are usually divided by (compared to) the diameter of the thickest part of the fuselage so that rockets of different size can be compared: this diameter is therefore one Caliber.

Centre of Pressure (CP):
The point on the rocket's surface where the average of all the aerodynamic forces from the nose, body, and fins act. This must be behind the Centre of Gravity (CG) by at least one **Caliber.**

Roll resonance:
Fin-stabilised rockets have a frequency at which they will nod back and forth in pitch in response to, say, a gust of wind. This 'weathercock' frequency changes with airspeed.

If the fins are misaligned and cause the vehicle to spin along its (roll) long axis, then the spin angular frequency (roll rate times 2 pi) might briefly equal the weathercock frequency at some point in the flight, usually at the low airspeeds just after launch, and if the **Barrowman** stability is too low. This causes a corkscrewing trajectory that may later suddenly stabilise pointed in any direction.
On military test-ranges it's been found that liquid mass onboard a rapidly spinning finned projectile makes the vehicle much less stable, so nitrous compounds the problem.
So don't misallign the fins: spin stabilisation was a '60's fashion that won't pay off.

References

Ref. 1: 'Fluid oscillations in the containers of a space vehicle and their influence upon stability'
by Helmut F. Bauer, NASA technical report TR R-187, Feb. 1964

Ref. 2: TRW Space Data book, 4th edition, Figure 6.A4, sloshing data,
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Ref. 3: 'Wind effects on launch vehicles', Section 4-4-3 propellant dynamics
The advisory group for research and development, NATOAGARDograph 115,
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Ref.4: 'Moment of inertia and damping of liquids in baffled cylindrical tanks' by Dodge and
Kana, NASA contractor report CR-383 (available on the web)